



The Mandelbrot Competition

Round Five Test

Time Limit:
40 minutes

Name: _____

<p>1. Determine the value of $\frac{1 + 2 + 3 + \dots + 19}{2 + 3 + 4 + \dots + 20}$, written as a reduced fraction.</p>		①									
<p>2. Orlando is culturing cells in a biology lab. The cell population grows exponentially, doubling in size every six hours. If there are currently about 800 cells, then which is the best estimate of the number of cells three hours later? A) Less than 1180. B) Between 1180 and 1220. C) More than 1220.</p>		①									
<p>3. The arrow maze at right is traversed as follows. Begin in square 7, and at each step move in the direction given by the arrow in your square to an adjacent square, then rotate the arrow in the square you just left 90° clockwise. Continue moving until you exit the maze by landing outside the grid. Which square is the last one you visit before exiting?</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td style="text-align: center;">1 ↑</td> <td style="text-align: center;">2 →</td> <td style="text-align: center;">3 ↓</td> </tr> <tr> <td style="text-align: center;">4 →</td> <td style="text-align: center;">5 ↑</td> <td style="text-align: center;">6 ↓</td> </tr> <tr> <td style="text-align: center;">7 ↑</td> <td style="text-align: center;">8 ←</td> <td style="text-align: center;">9 ←</td> </tr> </tbody> </table>	1 ↑	2 →	3 ↓	4 →	5 ↑	6 ↓	7 ↑	8 ←	9 ←	②
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<p>4. For most values of b the equation $3(7 + x) = b(7 - x)$ has a solution. For example, when $b = 11$ we can solve to find $x = 4$. Find the only value of b for which this equation has no solution.</p>		②									
<p>5. Let us say that a finite sequence a_1, a_2, \dots, a_n of n integers is <i>thrilling</i> if the sum of any two consecutive terms of the sequence is a power of three. (The powers of three are 1, 3, 9, 27, ...) What is the length of the shortest thrilling sequence whose first term is 1 and whose last term is 100?</p>		②									
<p>6. Mr. Strump has formed three person groups in his math class for working on projects. Every student is in exactly two groups, and any two groups have at most one person in common. In fact, if two groups are chosen at random then the probability that they have exactly one person in common is one-third. How many students are there in Mr. Strump's class?</p>		③									
<p>7. In triangle ABC point M is the midpoint of \overline{AB}, point N is on side \overline{BC} (but is not the midpoint), and segments \overline{AN} and \overline{CM} intersect at P. If we have $PA = 7$, $PC = 5$, and $PN = 3$ then compute length PM.</p>		③									

SCORE:

